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QUANTIFYING THE QUALITY OF PARTIAL MODEL COUPLING AND ITS EFFECT ON THE SIMULATED STRUCTURAL BEHAVIOR

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Abstract. *The process of analysis and design in structural engineering requires the consideration of different partial models, for example loading, structural materials, structural elements, and analysis types. The various partial models are combined by coupling several of their components. Due to the large number of available partial models describing similar phenomena, many different model combinations are possible to simulate the same aspects of a structure. The challenging task of an engineer is to select a model combination that ensures a sufficient, reliable prognosis. In order to achieve this reliable prognosis of the overall structural behavior, a high individual quality of the partial models and an adequate coupling of the partial models is required.*
Several methodologies have been proposed to evaluate the quality of partial models for their intended application, but a detailed study of the coupling quality is still lacking. This paper proposes a new approach to assess the coupling quality of partial models in a quantitative manner. The approach is based on the consistency of the coupled data and applies for uni- and bidirectional coupled partial models. Furthermore, the influence of the coupling quality on the output quantities of the partial models is considered.
The functionality of the algorithm and the effect of the coupling quality are demonstrated using an example of coupled partial models in structural engineering.

1 INTRODUCTION

The models used in structural engineering to design for serviceability and the ultimate limit state are composed of several partial models (PM) and their couplings (C). A partial model describes a component of the global model, e.g. loading, material, or the level of abstraction. For each class of PMs, e.g. the material behavior of steel, several possibilities of modeling are available. If the material model is relevant for the structural behavior, the structural engineer needs to decide, whether a linear or a non-linear material model should be used and whether further effects, e.g. long-term behavior, have to be considered. Apart from the selection of appropriate partial models the coupling of the individual PMs is a key issue. Some partial models might interact with each other, thus a coupling is substantial and the quality of this coupling influences the quality of the global model.

In recent years, strategies to estimate the quality of partial models, [1], [2], and to quantify the influence of the partial models on the global model prognosis [3] have been developed. Furthermore, the quantification of the prognosis quality of a global model, neglecting the influence of coupling quality, is described in [3]. The assessment of software coupling has been shown in [4], but does not apply to partial models directly. Altogether, the evaluation of partial model coupling and its influence on the prognosis of a global model has not been addressed so far.

In the scope of this paper a method to quantify the quality of data coupled partial models is presented. The basis of the procedure is the consistency of data belonging to the coupled partial models. Besides the pure data integrity the influence of the coupling on the partial models' output is taken into account within the framework of the evaluation algorithm.

In the next section some basic principles and methods are introduced. Section three presents the method of evaluation and section four gives an example of coupling quality evaluation. Finally, conclusions are drawn.

2 BASIC METHODS AND PRINCIPLES

2.1 Graphical Representation of Coupled Partial Models

Global models used in engineering consist of several partial models. Figure 1 depicts a structure of a simply supported beam, connected to a clamped column with a footing. On the left side the overall structure is presented all in one, on the right side the structural parts are decoupled. The partial models that are exemplarily depicted are: load models for dead load, live load, and wind load, material models, models of geometric non-linear kinematics, and soil.

Stein, Lahmer, and Bock [5] show that a global model can be represented schematically by a graph, consisting of vertices – symbolizing the partial models – and edges – symbolizing the coupling. This idea is extended within the scope of this paper. The global model in Figure 1, represented by the graph in Figure 2, is separated into its structural components; beam, column, and foundation. Due to the numerical calculation, a discretization of the structural parts is necessary, for example using 1D-beam or 3D-volume elements. Each of the structural parts consist of several classes of partial models, which are arranged according to the sequence of the analysis. These classes of partial models may include several different representations – partial models - of a phenomenon, for example material behavior. Only one partial model of a class can be used at the same time when modeling a global system.

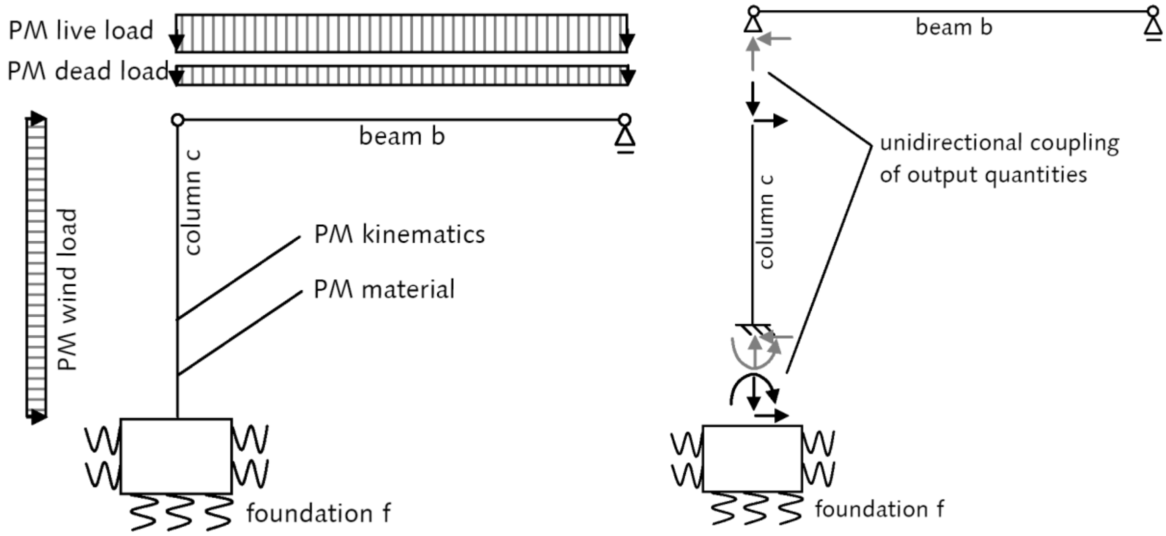

Figure 1: System of coupled partial models (PM), complete coupled system (left), decoupled system (right)

Within Figure 2 the coupling of partial models or structural parts is illustrated by arrows. These coupling are distinguished into unidirectional and bidirectional coupling [4]. If unidirectional coupling is applied, for example coupling beam and column, the output of the beam, in this case support forces, is the input of the column, but the output of the column, in this case deformations, is not the input of the beam. In case of bidirectional coupling, the output (deformations) of the column is also considered as input (pre-deformations of the support) of the beam. For some model combinations bidirectional coupling is essential, e.g. coupling of the material model. The output of the material model, the stresses, is required for the calculation of the column, thus backward coupling is substantial.

2.2 Sensitivity Analysis applied to Partial Models

Sensitivity analyses quantify the influence of input parameters on the output of a model. As proposed in [3], variance-based global sensitivity analysis can also be used to study the influence of partial models on the output of the global model. This procedure detects the most influential classes of the partial models. Consequently, when evaluating the quality of the global model, the individual quality of the partial models with high influences on the system's behavior is crucial for the overall prognosis quality. This algorithm to quantify the influence of classes of partial models is the basis of this investigation of coupling quality and is described in the following.

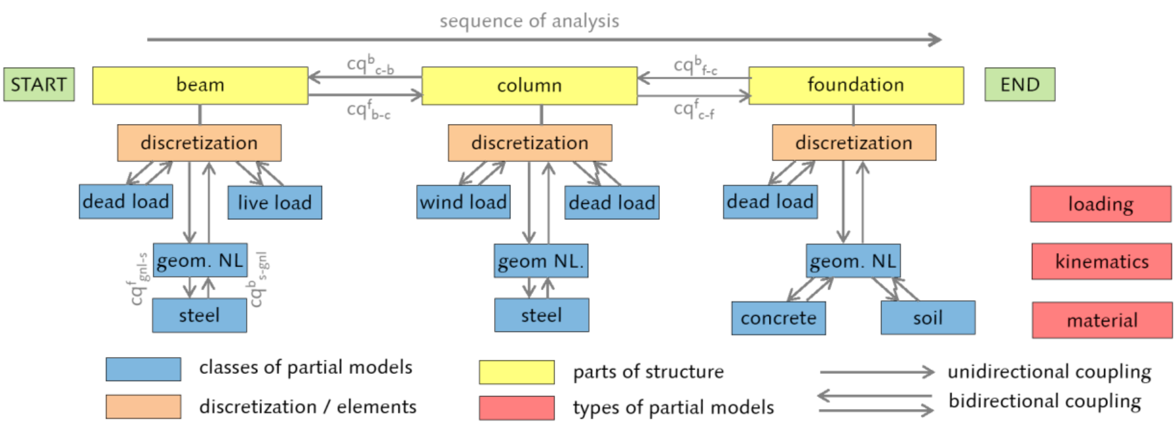

Figure 2: Representation of coupled partial models by a graph

Each of the classes of partial models *i, j* is represented by a uniformly distributed, discrete random parameter

$$X_i \in \{0,1\}, X_j \in \{0,1\}, \ldots \tag{1}$$

A value of $X_i=0$ denotes the deactivated class of partial models i, for example geometric non-linearity is not included, and $X_i=1$ denotes the activated class of partial models i. The global model Y is calculated for all possible combinations of the number of N_p partial model classes, which is in case of the discrete parameters a total of

$$N = 2^{N_p} \tag{2}$$

combinations. The first-order sensitivity index quantifies the exclusive influence of the parameter X_i and is defined as follows [6]:

$$S_i = \frac{V\left(E\left(Y \mid X_i\right)\right)}{V(Y)} = \frac{V_i}{V(Y)}. \tag{3}$$

Herein, $V(E(Y|X_i))$ is the variance of the expected value of the model response Y when conditioning to X_i and $V(Y)$ is the variance of the system response when all parameters vary simultaneously. If the sum of all S_i is close to one, the model is purely additive and no interactions of parameters exist. A sum smaller than one denotes that parts of the variance cannot be explained when the interactions of parameters or coupling effects are neglected.

In order to take into account coupling effects, the total-effects sensitivity index S_{Ti} was introduced [7]

$$S_{Ti} = 1 - \frac{V\left(E\left(Y \mid \boldsymbol{X}_{\sim i}\right)\right)}{V(Y)}, \tag{4}$$

with the variance of the expected value $V(E(Y|\boldsymbol{X}_{\sim i}))$ for the case that all parameters but X_i itself are fixed, denoted as $\boldsymbol{X}_{\sim i}$. Besides the exclusive influence of the parameter X_i on the variance of the response, the S_{Ti} index considers the interaction of X_i with all other parameters $\boldsymbol{X}_{\sim i}$.

Differences among first-order and total-effects sensitivity indices indicate interaction of parameter X_i with all other parameters $\boldsymbol{X}_{\sim i}$. When using high-order indices these interactions can be directly apportioned to specific parameters/classes of partial models. The definition of the high-order index of parameter X_i and X_j is the following [8]:

$$S_{ij} = \frac{V\left(E\left(Y \mid X_i, X_j\right)\right) - V_i - V_j}{V(Y)} = \frac{V\left(E\left(Y \mid X_i, X_j\right)\right)}{V(Y)} - S_i - S_j, \tag{5}$$

wherein $V(E(Y|X_i,X_j))$ is the variance of the expected value of Y when conditioning to X_i and X_j simultaneously. High-order indices can be calculated for all combinations of input parameters. Summing up all high-order indices of a single variable results in the total-effects indices.

In the present case of discrete input parameters all first-order, total-effects, and high-order indices can be calculated directly from the results of model Y for the N combinations of input parameters without the usual need of specific sensitivity estimators, which require high computational effort.

3 COUPLING QUALITY

3.1 Quality of Data Coupling

Within the scope of this paper, coupling is defined as data coupling and the quality of coupling is related to the quality of data transfer. Let α and β be quantities appearing in both partial models k and l at the same point on the structure, for example forces or displacements. A perfect data coupling ensures consistent data in both models, e.g. $\alpha^k=\alpha^l$, which refers to data coupling quality of $cq^f_{\alpha,k\text{-}l}=1$. The index f denotes the forward coupling according to the sequence of partial models within the graph, whereas b denotes the backward-coupling, for example $cq^b_{\beta,l\text{-}k}$. As the differences in transferred data increases, the quality of the coupling decreases down to a quality of zero when no data is transferred. This leads to the following definition of data coupling quality:

$$cq^{\mathrm{f}}_{\alpha,k-l} = 1 - \frac{\left|\alpha^k - \alpha^l\right|}{\max\left\{\left|\alpha^k\right|,\left|\alpha^l\right|\right\}} \quad \text{and} \quad cq^{b}_{\beta,l-k} = 1 - \frac{\left|\beta^k - \beta^l\right|}{\max\left\{\left|\beta^k\right|,\left|\beta^l\right|\right\}}. \tag{6}$$

The data coupling quality depends on the quantity being compared. As a coupling might consist of numerous data, the mean quality of N_f forward and N_b backward transferred data is derived with

$$\overline{cq}^{f}_{k-l} = \sum_{\alpha=1}^{N_f} cq^{\mathrm{f}}_{\alpha,k-l} \quad \text{and} \quad \overline{cq}^{b}_{l-k} = \sum_{\beta=1}^{N_b} cq^{b}_{\beta,l-k}. \tag{7}$$

An example of coupling is the data transfer of the support forces of the column to the foundation in Figure 1, when both structural parts are analyzed seperately. The forward quantities normal force, shear force, and bending moment are transferred to the foundation, and the backward quantities deformation in vertical direction u_z and horizontal direction u_x as well as the rotation φ_y, that occur due to the flexibility of the soil, are transferred back to the column support and are considered pre-deformations of the column at the support.

3.2 Influence of Coupling on Data

Independent from the quality of data coupling, the question of the influence of coupling on the data needs to be answered. For this reason, variance-based sensitivity analysis according to Section 2 is applied. In the current section the sensitivity of the forward coupled data quantities with respect to the partial models is explored, which is in contrast to the usual algorithms used when the sensitivity of certain structural quantities of the global system is determined.

For this analysis the partial models need to be distinguished based on their position in the sequence of the analysis: partial models arranged before the investigated coupling, denoted as $PM{\le}k$, and models arranged after the investigated coupling, denoted as $PM{\ge}l$. If the coupling quality of column-foundation needs to be determined for the graph in Figure 2, $PM{\le}k$ refers to all models directly linked to the beam and the column, and $PM{\ge}l$ refers to all models directly linked to the foundation.

Using high-order indices, the influence of partial models on the transferred data can be apportioned to each model and to several groups of models. In the present case we are interested in the sensitivity of the transferred data with respect to all $PM{\le}k$ and all $PM{\ge}l$. The sum of high-order indices for the groups of models becomes

$$\sum_{PM \leq k} S_\alpha = \sum_{i \leq k} \sum_{j \leq k} S_{ij,\alpha} \quad \text{and} \quad \sum_{PM \geq l} S_\alpha = \sum_{i=1}^{N_p} S_{Ti,\alpha} - \sum_{PM \leq k} S_\alpha \ . \tag{8}$$

In $\sum_{PM \leq k} S_\alpha$ no first-order or higher-order indices referring to any *PM≥l* are included. This value is a measure of the importance of forward coupling for quantity α. In contrast to this, $\sum_{PM \geq l} S_\alpha$ indicates the importance of backward coupling and includes all first-order for *PM≥l* and all high-order terms referring to any *PM≥l.* The need for bidirectional coupling increases with an increasing influence of backward coupling. Hence, the coupling quality is more and more dependent on the quality of the backward coupling.

3.3 Quality of Partial Model Coupling

In order to derive the quality of PM coupling, the data coupling quality and the influence of coupling are combined. The final application of the derived coupling quality is the consideration of it within the framework of model evaluation, thus the quality of coupling is defined with this motive. In order to do so, the coupling quality depends on the position of the output quantity in the graph, for which the influence of coupling is investigated for.

When the coupling quality is evaluated for coupled PMs that are after the investigated output quantity in the sequence of the analysis, a backward coupling is essential; otherwise no information of the partial models arranged after the coupling can be transferred back to the PMs that are before in the sequence of the analysis. In this case, quality of coupling becomes

$$CQ_{k-l} = \overline{cq}^{f}_{k-l} \times \overline{cq}^{b}_{l-k} . \tag{9}$$

If one of the forward or backward data coupling quality is zero, the total quality of coupling becomes zero as well.

When coupling quality is evaluated for coupled PMs that are arranged before the investigated output quantity, the backward coupling might influence the coupled quantities to some extent, but it is not obligatory. In this case the quality is defined as

$$CQ_{k-l} = \frac{\sum_{\alpha=1}^{N_f} \left(cq^{f}_{\alpha,k-l} \sum_{PM \leq k} S_{\alpha_f} \right) + \overline{cq}^{b}_{l-k} \frac{1}{N_f} \sum_{\alpha=1}^{N_f} \left(\sum_{PM \geq l} S_\alpha \right)}{\sum_{PM} \overline{S}_{ij}} . \tag{10}$$

The forward data coupling quality cq^{f}_{α} is directly linked to the sensitivity indices of α. For backward data coupling quality this is not possible, because it cannot be determined which of the backward coupling quantities β has an influence on α. Furthermore, the number of forward and backward coupling quantities might differ. Hence, the mean value of sensitivity indices of α is multiplied with the mean of backward data coupling quality $\overline{cq}^{b}_{l-k}$.

4 EXAMPLE

4.1 Partial Models and First Results

In the following, an example depicted in Figure 3 is analyzed with respect to coupling quality. The considered partial models are: live load beam (PM1), non-linear material behavior

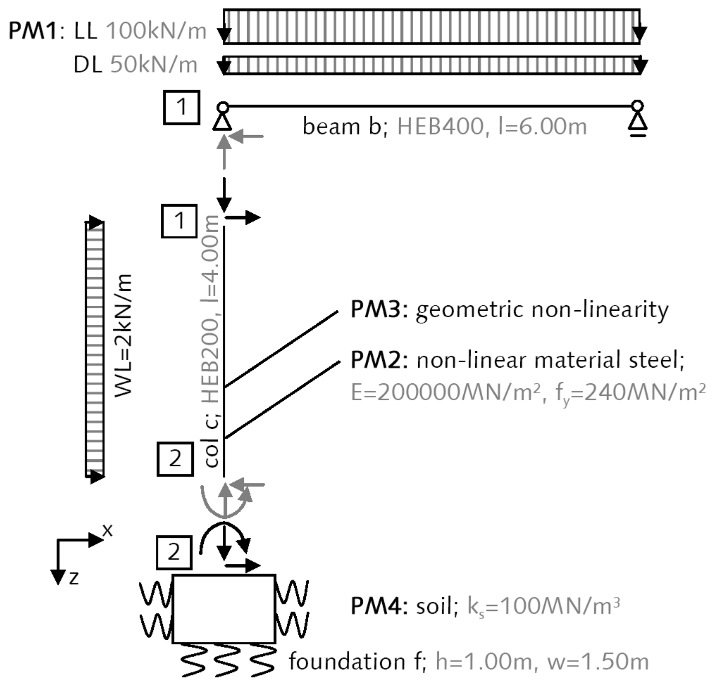

Figure 3: Example of coupled partial models

of the steel columns (PM2), geometric non-linear behavior of the steel column (PM3), and elastic behavior of soil (PM4). Coupling positions of the structural parts are between beam and column, denoted as 1, and between column and foundation, denoted as 2. Further parameters are depicted in the figure.

First, the system is calculated considering perfect model coupling and the resulting major forces, moment, displacements, and rotations of the three structural parts are given in Table 1. From these numbers the qualitative influence of the several classes of partial models is derived, e.g. the influence of geometric non-linearity PM3 on the bending moment at column support, $M_{y2,c}$.

Second, Table 2 shows the results of different couplings of the structural parts, distinguished into uni- and bidirectional coupling. Furthermore, bidirectional coupling with a limited number of iterations between the structural subsystems is given. From these numbers the relationship between the couplings are found. For example the support force of the beam $F_{z1,b}$ is independent from these couplings. This is in contrast to the support moment of the column $M_{y2,c}$, which depends on the type of column-foundation foundation.

X_{PM1}	X_{PM2}	X_{PM3}	X_{PM4}	$F_{z1,b}$ [kN]	$F_{z2,c}$ [kN]	$M_{y2,c}$ [kNm]	$u_{x1,c}$ [mm]	$\varphi_{y2,c}$ [E-3]	$\varphi_{y2,f}$ [E-3]
0	0	0	0	150	150	16.0	5.5	0.0	0.0
1	0	0	0	450	450	16.0	5.5	0.0	0.0
1	0	1	0	450	450	19.3	7.3	0.0	0.0
1	0	0	1	450	450	16.0	9.9	-0.86	-0.97
1	0	1	1	450	450	21.7	13.8	-1.06	-1.06
1	1	1	1	450	450	21.7	13.8	-1.06	-1.06

Table 1: Results for different model classes, perfect bidirectional coupling

coupling b-c	coupling c-f	$F_{z1,b}$ [kN]	$F_{z1,c}$ [kN]	$M_{y2,c}$ [kNm]	$M_{y2,f}$ [kNm]	$\varphi_{y2,c}$ [E-3]	$\varphi_{y2,f}$ [E-3]
unidirectional	unidirectional	450	450	19.3	19.3	0.00	-0.97
bidirectional	unidirectional	450	450	19.3	19.3	0.00	-0.97
unidirectional	bidirectional	450	450	21.7	21.7	-1.06	-1.06
bidirectional	bidirectional	450	450	21.7	21.7	-1.06	-1.06
bidirectional	bidirectional, only 1 iteration	450	450	21.5	21.5	-0.97	-1.05

Table 2: Results for different coupling types, all partial models considered

4.2 Influence of Partial Models

The influence of the partial models is determined by means of sensitivity analysis according to [3], applying a perfect data coupling. The resulting high-order sensitivity indices for selected output quantities are given in Table 3. The output $F_{z1,b}$ depends only on PM1 live load beam, thus no interaction effects with other PMs occur. Contradictory to this, $M_{y2,c}$ depends on several partial models and an interaction of these PMs is quantified by the high-order indices, for example an interaction of live load PM1 and geometric non-linearity PM3 with S_{13}=0.181. The quantity $M_{y2,c}$ depends also on the soil model PM4. This effect can only occur when backward coupling from the foundation to the column exists, thus a higher demand for this coupling is present, in contrast to the beam-column coupling.

	$F_{z1,b}$	$F_{z2,c}$	$M_{y2,c}$	$u_{x1,c}$	$\varphi_{y2,f}$
S_1	1.000	1.000	0.181	0.035	0.002
S_2	0.000	0.000	0.000	0.000	0.000
S_3	0.000	0.000	0.536	0.104	0.005
S_4	0.000	0.000	0.037	0.801	0.984
$S_{13}= S_{31}$	0.000	0.000	0.181	0.035	0.002
$S_{14}= S_{41}$	0.000	0.000	0.014	0.005	0.002
$S_{34}= S_{43}$	0.000	0.000	0.037	0.014	0.005
$S_{134}= S_{314}= S_{413}$	0.000	0.000	0.014	0.005	0.002
S_{T1}	1.000	1.000	0.389	0.081	0.007
S_{T2}	0.000	0.000	0.000	0.000	0.000
S_{T3}	0.000	0.000	0.768	0.158	0.013
S_{T4}	0.000	0.000	0.102	0.826	0.992
$\Sigma S_{PM\leq k}$	1.000	1.000	1.079	0.209	1.011
$\Sigma S_{PM\geq l}$	0.000	0.000	0.181	0.856	0.000
ΣS_T	1.000	1.000	1.260	1.065	1.011

Table 3: Sensitivity indices of specific model responses

coupling b-c	cq^f_{Fz1}	cq^b_{uz1}	$\sum_{PM\leq 1} S_{Fz1}$	$\sum_{PM\geq 2} S_{Fz1}$	$\frac{1}{2}\sum_{f=1}^{2}\left(\sum_{PM\geq 2} S_{\alpha_f}\right)$	$CQ^b_{b\text{-}c}$	$CQ^c_{b\text{-}c}$
unidirectional	1.00	0.00	1.00	0.00	0.00	0.00	1.00
bidirectional	1.00	1.00				1.00	1.00

Table 4: Results for coupling quality beam-column

coupling c-f	cq^f_{My2}	$cq^b_{\varphi y2}$	$\sum_{PM\leq 3} S_{My2}$	$\sum_{PM\geq 4} S_{My2}$	$\frac{1}{3}\sum_{f=1}^{3}\left(\sum_{PM\geq 4} S_{\alpha_f}\right)$	$CQ^c_{c\text{-}f}$	$CQ^f_{c\text{-}f}$
unidirectional	1.00	0.00	1.08	0.18	0.06	0.00	0.94
bidirectional	1.00	1.00				1.00	1.00
bidirectional, only 1 iteration	1.00	0.92				0.92	0.99

Table 5: Results for coupling quality column-foundation

4.3 Coupling Quality

Within this section the coupling quality is estimated considering all four partial models of the example. Different couplings of beam-column and column-foundation are investigated. The further couplings, e.g. the material behavior with the kinematics, do not provide any data loss and have a quality of one. The qualities for specific quantities are given in the Tables 4 and 5. As mentioned earlier, the quality of partial model coupling *CQ* depends on the quantity of interest, in particular on the position of the quantity of interest within the sequence of the analysis. Hence, *CQ* is calculated for the different involved partial models/structural parts, denoted for example as CQ^c for coupling quality of the column.

The coupling beam-column consists of two output quantities of the beam, $F_{z1,b}$ and $F_{x1,b}$, and two output quantities of the column, $u_{z1,b}$ and $u_{x1,c}$. The forward coupling quality is always one, as the output quantities of the beam are directly applied to the column and no data loss occurs. In case of unidirectional coupling the data coupling quality of the backward coupling is zero. Analyzing the sensitivity indices reveals that the output quantity $F_{z1,b}$ depends only on PM1, thus no backward coupling is necessary when *CQ* is analyzed for the column and this results to $CQ^c_{b\text{-}c}=1.0$ according to Eq. (10). When analyzing the quality for quantities of the beam according to Eq. (9), values of $CQ^b_{b\text{-}c}=0.0$ and $CQ^b_{b\text{-}c}=1.0$ for the unidirectional and bidirectional case are obtained. The zero coupling quality for unidirectional interaction results from necessity of backward coupling for the beam in order to take into account output quantities of the column.

The coupling column-foundation consists of three output quantities of the column, $F_{z2,c}$, $F_{x2,c}$ and $M_{y2,c}$, as well as three output quantities of the foundation, $u_{z2,f}$, $u_{x2,f}$ and $\varphi_{y2,f}$. The forward coupling quality is still always one and the backward coupling quality is always zero in case of unidirectional coupling. As already mentioned, $M_{y2,c}$ depends to some extent on PM4. This is pointed out when comparing the sum of the sensitivity indices of all $PM\leq 3$ before and all $PM\geq 4$ after the coupling, 1.08 and 0.18. Hence, the coupling quality of the partial models depends on the quality of the forward and backward coupling even for response quantities that are after the coupling. The data coupling quality of the support moment is given in Table 5. The resulting coupling qualities are also shown in this Table 5 for two response quantities: first belonging to the column $CQ^c_{c\text{-}f}$, which is before coupling and calculated according to Eq. (9), and second for the foundation $CQ^f_{c\text{-}f}$, which is after the coupling analyzed according to

Eq. (10). The value of $CQ^{c}_{c\text{-}f}$ is zero in the unidirectional case, as no information of PM4 can be transferred back to the column. When analyzing $CQ^{f}_{c\text{-}f}$ it is observed that a unidirectional coupling still leads to a quality of $CQ^{f}_{c\text{-}f}$=0.94, as the output quantities of the column are mainly defined by the forward coupling and only relatively small parts of the output quantities are influenced by the backward coupling. In case of bidirectional coupling with only one iteration between the structure of the column and foundation, a high value of $CQ^{f}_{c\text{-}f}$=0.99 is determined, thus one iteration already gives satisfying results.

5 CONCLUSIONS

This paper presented a method to calculate data coupling quality and to quantify the influence of coupling on the output data in the case of coupled partial models. By doing so the determination of coupling quality of partial models in the context of a global system is acconted for.

The method provides a useful tool to determine the necessity to couple partial models in a uni- or bidirectional manner. Hence, the algorithm allows for a reduction of complexity of global systems when bidirectional coupling is less important. Furthermore, the understanding of the system's behavior increases when the results of the method are analyzed.

The defined coupling quality can be considered within a framework of model evaluation in order to provide a total measure for the quality of coupled partial models. This global measure should take into account the influence of partial models on the global response, the quality of the partial models, and the quality of their coupling.

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